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A Generalized Ballistic Force System

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DEPARTMENT OF THE ARMY PROJECT No. 5B0303001
ORDNANCE RESEARCH AND DEVELOPMENT PROJECT No. TB3-0108

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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ABSTRACT

The dependence of the aerodynamic force and moment acting on a missile in free flight on the past history of the missile's motion has been omitted from the Ballistic Theory and only partly considered in the Aerodynamic Theory. In this study the linear terms arising from a consideration of the forces and moments due to acceleration effects are added to the theory of motion. The resulting equations of motion and their solutions yield a more physically complete system, useful to both ballisticians and aerodynamicists in understanding the free flight performance of symmetrical missiles. The expansion of the transverse force and moment for a missile possessing a plane of mirror symmetry is given in an appendix.

*Bureau of Ordnance, Department of the Navy.

INTRODUCTION

The aerodynamic force and moment which act on a missile in free flight are assumed to depend on the attitude of the missile with respect to its air velocity vector and on the motion of the missile. For this reason the usual coordinate systems with axes numbered 1, 2, 3 are located on the missile so that their 1-axes coincide with the missile's axis of rotational symmetry. If $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$ and $\vec{\Omega} = (\Omega_1, \Omega_2, \Omega_3)$ are the angular velocities of the missile and of the coordinate system with respect to an inertia system then $\omega_2 = \Omega_2$ and $\omega_3 = \Omega_3$. The coordinate system can then be completely determined by the specification of Ω_1 and an initial orientation of the 2-axis. There are at least three different choices which are at present in use. They are:

1. Missile-fixed coordinates[1] ($\Omega_1 = \omega_1$) which are used by aerodynamicists for the specification of the force system and for studies of the implications of symmetry on the aerodynamic force and moment;

2. Kelley-McShane or non-rolling coordinates[2] ($\Omega_1 = 0$ and 2 axis initially orientated in the horizontal plane) which are used in their theory of the yawing motion of symmetric missiles; and

3. Fixed-plane coordinates (2 axis initially in the horizontal plane and Ω_1 so selected that it stays there) which are convenient for free flight spark range work.*

It is usually assumed for small yaws that the aerodynamic force and moment, which are functions of the linear velocity $\vec{u} = (u_1, u_2, u_3)$ and the angular velocity $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$ of the missile, are linear functions of their cross components, u_2, u_3, ω_2 and ω_3. Since the ballistician mainly deals with missiles possessing rotational symmetry this symmetry is exploited by the use of complex variables. The cross velocity and cross angular velocity may be written in nondimensional form as $\lambda = \frac{u_2 + iu_3}{u_1}$ and $\mu = \frac{(\omega_2 + i\omega_3)d}{u_1}$ where d is the missile's diameter. For missiles having mirror symmetry and trigonal** or greater rotational symmetry, the aerodynamic force, $\vec{F}$, and moment, $\vec{M}$, are given by the following equations [1, 2, 4]:

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* Ω_1 for these coordinates differs from zero by quantities containing the square of the yaw and is zero for planar yawing motion. A correction for spark range measurements is given in Ref. 3.

** A missile possesses trigonal rotational symmetry if it possesses an angle of rotational symmetry equal to $\frac{2\pi}{3}$.

$$F_1 = - \rho d^2 u_1^2 K_{DA} \quad (1)$$

$$F_2 + iF_3 = \rho d^2 u_1^2 \left[(- K_N + i \nu K_F) \lambda + (\nu K_{XF} + iK_S)\mu \right] \quad (2)$$

$$M_1 = - \rho d^3 u_1^2 \nu K_A \quad (3)$$

$$M_2 + iM_3 = \rho d^3 u_1^2 \left[(-\nu K_T - iK_M) \lambda + (- K_H + i\nu K_{XT})\mu \right] \quad (4)$$

where ρ is the air density

$\nu = \frac{\omega_1 d}{u_1}$ is dimensionless spin and the K's are ballistic coefficients defined by the above equations. Note that since Eqs. (2) and (4) are relations between two-dimensional vectors the equations are independent of the selection of Ω_1 and thus are valid for the three coordinate systems.

Although this ballistic force system has seemed perfectly satisfactory to the ballistician, the aerodynamicist [5, 6] has worked for some time with terms involving the time derivative of λ as well as terms in λ and μ. These additional derivative terms are measures of the past history of the the missile's motion or lags in the flow development about the missile. As we shall show, the inclusion of these new terms and the requirement of consistent center of mass transformation require the introduction of additional terms in the time derivative of μ. These terms in the derivative of μ also appear quite naturally in Ref. 8 and 9 where slender body values for all non-Magnus coefficients are calculated.

In this report we shall introduce two new variables which are related to the cross acceleration and cross angular acceleration and investigate their effect on the yawing motion, the swerving motion, and the center of mass transformation. Since this generalized ballistic force system has a one-to-one correspondence to the complete aerodynamic system, the relations between them as well as the slender body values for the non-Magnus coefficients can and will be stated.

THE NEW VARIABLES

The linear acceleration vector, $\vec{a}$, and angular acceleration vector, $\vec{\alpha}$, can be expressed in terms of the linear and angular velocity vectors by the relations:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \mathring{u}_1 \\ \mathring{u}_2 \\ \mathring{u}_3 \end{bmatrix} + \begin{bmatrix} \Omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \times \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \mathring{u}_1 + \omega_2 u_3 - \omega_3 u_2 \\ \mathring{u}_2 + \omega_3 u_1 - \Omega_1 u_3 \\ \mathring{u}_3 + \Omega_1 u_2 - \omega_2 u_1 \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \mathring{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} + \begin{bmatrix} \Omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \times \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} \mathring{\omega}_1 \\ \mathring{\omega}_2 + \omega_3\omega_1 - \Omega_1\omega_3 \\ \dot{\omega}_3 + \Omega_1\omega_2 - \omega_2\omega_1 \end{bmatrix} \qquad (6)$$

The dimensionless cross linear and cross angular acceleration vectors can be written as

$$\frac{(a_2 + ia_3)d}{u_1^2} = \frac{(\dot{u}_2 + i\dot{u}_3)d}{u_1^2} - i\mu + i\frac{\Omega_1 d}{u_1}\lambda$$

$$= \lambda' + \frac{u'_1}{u_1}\lambda - i\mu + i\frac{\Omega_1 d}{u_1}\lambda \qquad (7)$$

$$\frac{(\alpha_2 + i\alpha_3)d^2}{u_1^2} = \frac{(\mathring{\omega}_2 + i\mathring{\omega}_3)d^2}{u_1^2} - i\nu\mu + i\frac{\Omega_1 d}{u_1}\mu$$

$$= \mu' + \frac{u'_1}{u_1}\mu - i\nu\mu + i\frac{\Omega_1 d}{u_1}\mu \qquad (8)$$

where primes indicate derivatives with respect to non-dimensional arc length $p = \int_0^t \frac{u_1}{d}\, dt$.

Our first choice for the new variables would be the dimensionless cross linear and cross angular accelerations. But an examination of Eqs. (7 - 8) shows a basic handicap in this selection. If we did take these to be our additional variables in the force and moment expansions, our first step in deriving the equations of motion would be to make use of Eqs. (7 - 8) to obtain equations of motion in λ, μ, λ', and μ'. But then coefficients of λ and μ would be unnecessarily complicated by contributions from the coefficients of the new variables!

Since we would prefer to retain the convenient form of Eqs. (1 - 4), an important feature of any selection for new variables should be that they be two dimensional vectors. In order to insure this property, a closer examination of our idea of a vector is in order. By a vector we mean a quantity for which certain operations are defined (multiplication by scalar, addition and subtraction, vector and scalar products) and which is represented in a coordinate system by a set of numbers which transform in a prescribed manner under a coordinate system transformation. In particular, if $\begin{pmatrix} x_2 \\ x_3 \end{pmatrix}$ and $\begin{pmatrix} \hat{x}_2 \\ \hat{x}_3 \end{pmatrix}$ are two representations of the same two-dimensional vector in Cartesian coordinate systems which differ by a rotation through the angle θ,

$$\begin{pmatrix} \hat{x}_2 \\ \hat{x}_3 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \end{pmatrix} \tag{9}$$

In our complex form this can be written as

$$\hat{z} = z e^{i\theta}, \tag{10}$$

where $\hat{z} = \hat{x}_2 + i\hat{x}_3$ and $z = x_2 + ix_3$ are different representations of the same vector quantity.

In Eqs. (7) and (8) we see that the quantities λ' and μ' appear. These quantities are complex numbers whose real and imaginary parts are the derivatives of the components of two dimensional vectors. A natural question to ask is whether these complex numbers also represent two dimensional vectors. If we differentiate Eq. (10) and remember that θ is not necessarily constant,

$$\hat{z}' = (z' + i\theta' z) e^{i\theta} \tag{11}$$

Thus we see that λ' and μ' do not in general represent vectors.

A possible objection to the above statement might be based on the observation that by means of Eqs. (7 - 8), λ' and μ' can be written as linear combinations of vectors and, therefore, they must represent vectors themselves. Now a linear combination of vectors is a summation of vectors multiplied by scalars where scalars are quantities which are represented in a coordinate system by a number which is invariant under coordinate system transformations. Ω_1, which appears in Eqs. (7 - 8), is defined to be the axial component of the angular velocity with respect to an inertia system of the coordinate system in which these equations are expressed, and, hence, can change under a coordinate system transformation. (Note that the only coordinate transformations which are allowed are given by Eq. (10).) Thus we see that λ' and μ' are not really equal to linear combinations of vectors.

If we rearrange Eqs. (7) and (8),

$$\lambda' + i\frac{\Omega_1 d}{u_1}\lambda = \frac{(a_2 + ia_3)d}{u_1^2} + i\mu - \frac{u_1'}{u_1}\lambda \tag{12}$$

$$\mu' + i\frac{\Omega_1 d}{u_1}\mu = \frac{(a_2 + ia_3)d^2}{u_1^2} + \left(i\nu - \frac{u_1'}{u_1}\right)\mu \tag{13}$$

Here the quantities on the left sides of Eqs. (12 - 13) are vectors because they are equal to linear combinations of vectors. We also notice that these quantities reduce to λ' and μ' respectively in non-rolling coordinates which are equivalent to the fixed-plane coordinates for small yaw ($\Omega_1 \doteq 0$). For this reason we will call them fixed-plane derivatives.

The aerodynamicist who usually deals with configurations without rotational symmetry prefers to make use of the missile-fixed coordinates, and, therefore, makes use of derivatives in those coordinates. If we subtract $i\nu\lambda$ and $i\nu\mu$ from both sides of Eqs. (12) and (13) respectively we would have these missile-fixed derivatives:

$$\lambda' + i\left(\frac{\Omega_1 d}{u_1} - \nu\right)\lambda = \frac{(a_2 + ia_3)d}{u_1^2} + i\mu - \left(\frac{u_1'}{u_1} + i\nu\right)\lambda \tag{12}$$

$$\mu' + i\left(\frac{\Omega_1 d}{u_1} - \nu\right)\mu = \frac{(\alpha_2 + i\alpha_3)d^2}{u_1^2} - \frac{u_1'}{u_1}\mu \tag{13}$$

In the table we show how these two sets of variables, fixed-plane and missile-fixed derivatives, appear in these two coordinate systems.

TABLE I

	Fixed-Plane Coordinates ($\Omega_1 \doteq 0$)	Missile-Fixed Coordinates ($\Omega_1 = \omega_1$)	
$\lambda' + i\frac{\Omega_1 d}{u_1}\lambda$	λ'	$\lambda' + i\nu\lambda$	fixed-plane derivatives
$\mu' + i\frac{\Omega_1 d}{u_1}\mu$	μ'	$\mu' + i\nu\mu$	
$\lambda' + i\left(\frac{\Omega_1 d}{u_1} - \nu\right)\lambda$	$\lambda' - i\nu\lambda$	λ'	missile-fixed derivatives
$\mu' + i\left(\frac{\Omega_1 d}{u_1} - \nu\right)\mu$	$\mu' - i\nu\mu$	μ'	

The first observation one can make from this table is that for the usual cases with which with aerodynamicist deals, namely where the spin is small ($\nu \ll 1$), the differences between the two definitions are small terms of second order and can be neglected. For rapid spin the case is quite different. Since the treatment of rapidly spinning models which do not possess rotational symmetry is very difficult and as far as the authors know has not as yet been completely considered, we will restrict ourselves to missiles possessing trigonal or greater symmetry. This is the case in which the ballistician is most interested and for this case the fixed-plane derivatives possess two advantages:

1. Almost all aerodynamic force and moment measurements made in either wind tunnels or free flight ranges are essentially made in fixed-plane coordinates. As can be seen from the table if we made use of missile-fixed derivatives, the non-derivative coefficients would be modified and unnecessarily complicated by contributions from the coefficients of these new terms. This objection is quite similar to the one we raised against the use of the cross linear and cross angular accelerations.

2. In a theoretical calculation of the aerodynamic coefficients for a body of revolution the assumption of no viscosity is usually made. Since in this case the air has no way of knowing whether the missile is spinning or not, the expansion of the force and moment in a coordinate system which is not rotating with respect to the air (fixed-plane coordinates) should not contain spin dependent terms. If we consider, for example, the expansion in fixed-plane coordinates of the cross force in terms of λ and its missile-fixed derivative, then

$$F_2 + iF_3 = C_1 \lambda + C_2(\lambda')_{\text{missile-fixed}} \tag{14}$$

From Table I it follows that

$$F_2 + iF_3 = C_1\lambda + C_2(\lambda' - i\nu\lambda) = (C_1 - i\nu C_2)\lambda + C_2\lambda' \tag{14'}$$

where C_1 and C_2 are complex functions of the aerodynamic coefficients. We know from the above considerations that $(C_1 - i\nu C_2)$ and C_2 can not be functions of spin and, hence, C_1 must vary linearly with spin. Similar observations apply to the transverse components of the moment and to the coefficients of μ and its missile-fixed derivative. But this means that a theoretical development for a body of revolution based on the assumption of no viscosity will contain non-zero Magnus-like coefficients! The fixed-plane derivatives avoid this difficulty.*

We, therefore, select $\lambda' + i\frac{\Omega_1 d}{u_1}\lambda$ and $\mu' + i\frac{\Omega_1 d}{u_1}\mu$ to be our new variables in the linear expansion of the aerodynamic force and moment. In order to consider implications of symmetry, we have to express the force and moment expansion in the missile-fixed coordinates**

$$\left(\frac{\Omega_1 d}{u_1} = \nu\right).$$

* In Ref. 8 Sacks used missile-fixed coordinates in which to calculate slender body coefficients but specifically defines α as a fixed-plane derivative. Strangely enough he does take $\dot{q}$ to be a missile-fixed derivative and, therefore, obtains non-zero values of $C_{Y_{pq}}$ and $C_{n_{pq}}$ for bodies of revolution.

** In the Appendix we develop the expansion of transverse force and moment for a missile with a plane of mirror symmetry but no rotational symmetry.

In the missile-fixed coordinate system these variables become $\lambda' + i\nu\lambda$ and $\mu' + i\nu\mu$. Now $\lambda' + i\nu\lambda$ and $\mu' + i\nu\mu$ both transform under the symmetry transformation exactly as λ and μ respectively and hence the form of the dependence of the force and moment on the new variables should be the same as that of λ and μ. We need only write the generalized form of Eqs. (2) and (4) and will use the subscript A for acceleration:

$$F_2 + iF_3 = \rho d^2 u_1^2 \Big[(- K_N + i\nu K_F)\lambda + (\nu K_{XF} + iK_S)\mu + (- K_{NA} + i\nu K_{FA}) (\lambda' + i\frac{\Omega_1 d}{u_1} \lambda) + (\nu K_{XFA} + iK_{SA}) (\mu' + i\frac{\Omega_1 d}{u_1} \mu)\Big] \tag{15}$$

$$M_2 + iM_3 = \rho d^3 u_1^2 \Big[(- \nu K_T - iK_M)\lambda + (- K_H + i\nu K_{XT})\mu + (- \nu K_{TA} - iK_{MA}) (\lambda' + i\frac{\Omega_1 d}{u_1} \lambda) + (-K_{HA} + i\nu K_{XTA}) (\mu' + i\frac{\Omega_1 d}{u_1} \mu)\Big] \tag{16}$$

The dependence of the force and moment on a_1 and α_1 is absorbed by the K's.

For the dynamic equations derived in this report we will work in the non-rolling coordinate system ($\Omega_1 = 0$) and Eqs. (15) and (16) will become less complicated. As has been mentioned before our "fixed-plane derivatives" are actually non-rolling derivatives and are only equal to fixed-plane derivatives for small yawing motion. Eqs. (17 - 20) of the next section are exactly true for any size of yaw when they are considered to be in the non-rolling system. We make use of the "fixed-plane coordinates" in this report mainly because they are easier to visualize.

EQUATIONS OF YAWING MOTION

For an arbitrary force system the equations of yawing motion for $\Omega = 0$ may be written in the form [2, 10]:

$$\frac{u_1'}{u_1} + \frac{i(\mu\bar{\lambda} - \bar{\mu}\lambda)}{2} = \frac{F_1 d}{m u_1^2} + J_g \tag{17}$$

$$\lambda' + \frac{u_1'}{u_1}\lambda - i\mu = \frac{(F_2 + iF_3)d}{m u_1^2} + J_g\lambda + \gamma \tag{18}$$

$$\nu' + \frac{u_1'}{u_1}\nu = \frac{k_1^{-2} M_1}{m u_1^2} \tag{19}$$

$$\mu' + \frac{u_1'}{u_1}\mu - i\bar{\nu}\mu = \frac{k_2^{-2}(M_2 + iM_3)}{m u_1^2} \tag{20}$$

where m = mass

$$J_g = \frac{g_1 d}{u_1^2}$$

$$\gamma = \frac{(g_2 + ig_3)d}{u_1^2} - J_g\lambda$$

(g_1, g_2, g_3) vector of acceleration due to gravity

k_1 axial radius of gyration in calibers

k_2 transverse radius of gyration in calibers

$$\bar{\nu} = \frac{k_1^2}{k_2^2}\,\nu$$

and $\bar{\lambda}$, $\bar{\mu}$ are complex conjugates of λ and μ.

If we introduce the usual definition $J_i = \frac{\rho d^3}{m} K_i$, assume that $K_{DA} \doteq K_D$ where K_D is the trajectory drag coefficient, and neglect the second order term $(\mu\bar{\lambda} - \bar{\mu}\lambda)$ in Eq. (17), Eqs. (17) and (19) can be written in the form

$$\frac{u_1'}{u_1} = - J_D + J_g \tag{21}$$

$$\nu' = (D - J_g)\nu \tag{22}$$

where $D = J_D - k_1^{-2} J_A$.

Substituting Eqs. (15), (16), and (21) in Eqs. (18) and (20), with $\Omega_1 = 0$ and defining $K_L = K_N - K_D$,

$$\begin{aligned} &\lambda' (1 + J_{NA} - i\nu J_{FA}) + \lambda(J_L - i\nu J_F) + \mu'(-\nu J_{XFA} - iJ_{SA}) \\ &\quad + \mu\left[(-\nu J_{XF} - i(1 + J_S)\right] - \gamma = 0 \end{aligned} \tag{23}$$

$$\begin{aligned} &\lambda' k_2^{-2} (\nu J_{TA} + iJ_{MA}) + \lambda k_2^{-2}(\nu J_T + iJ_M) + \mu'\left[1 + k_2^{-2}(J_{HA} - i\nu J_{XTA})\right] \\ &\quad + \mu\left[k_2^{-2} J_H - J_D + J_g - i\bar{\nu}(1 + k_1^{-2} J_{XT})\right] = 0 \end{aligned} \tag{24}$$

In order to eliminate μ and μ', we now operate on Eq. (23) with

$$\left\{\left[1 + k_2^{-2} (J_{HA} - i\nu J_{XTA})\right] \frac{d}{dp} + k_2^{-2} J_H - J_D + J_g - i\bar{\nu}(1 + k_1^{-2} J_{XT})\right\},$$

on Eq. (24) with $\left\{\left[\nu J_{XFA} + iJ_{SA}\right]\frac{d}{dp} + \left[\nu J_{XF} + i(1 + J_S)\right]\right\}$, and add. Assuming that derivatives of coefficients can be neglected and using (18) to eliminate ν', the result reduces to

$$(H_A - i\bar{\nu}\, X_A)\lambda'' + \left[\hat{H} + J_g - i\bar{\nu}\hat{X}\right]\lambda' + \left[-\hat{M} - i\bar{\nu}\,\hat{T}\right]\lambda = \hat{G} + \bar{\nu}(D - J_g)E \qquad (25a)$$

$$\bar{\nu}' = (D - J_g)\bar{\nu} \qquad (25b)$$

where the eight new symbols are defined in Table II.

The upper case letters with the exception of G and E are selected in order to identify the moment coefficient which is the principal constituent. The quite formidable expressions above may be simplified by certain quite reasonable size assumptions. We assume that $|J_F| < 10^{-4}$, $|J_i| < 3 \times 10^{-3}$ otherwise, $1 < \frac{B}{A} < 10^2$, $k_1^{-2} < 10$, $k_2^{-2} < 2$, $|\nu| < 1$, and $|\lambda| < .2$. Since $\frac{\rho d^3}{m}$ is usually about 5×10^{-5}, this restricts the magnitude of K_F to less than 2 and that of the other K_i's to less than 60. The requirement for the special case of J_g reduces to $d^{1/2} < (10^{-2}\ u)$ where d is in feet and u is ft/sec. From Eq. (23), it can be seen that μ is comparable with $- i\lambda'$, μ' with $- i\lambda''$ and hence the second term on the right side of Eq. (25) can be neglected. H_A and X reduce to unity and similar approximations apply to the other terms*

$$\therefore\ \lambda'' + (\hat{H} + J_g - i\bar{\nu})\lambda' + (- M - \bar{\nu}\ T)\lambda = G \qquad (26)$$

where $\hat{H} = H - k_2^{-2}\ (J_{MA} + \nu^2 k_1^{\ 2}\ J_{FA}) = J_L - J_D + k_2^{-2}(J_H - J_M - \nu^2 k_1^{\ 2}\ J_{FA})$

$\hat{M} = M = k_2^{-2}(J_M + \nu^2 k_1^{\ 2}\ \ J_F)$

$\hat{T} = T = J_L - k_1^{-2}\ \ J_T$

$\hat{G} = G = \gamma' - \left[(J_D - k_2^{-2}\ J_H - J_g) + i\bar{\nu}\right]\gamma$

and H, M, T, G are quantities defined in the yaw equation in Ref. 10 based on the unmodified ballistic force system.

* If we multiply (25a) by $(H_A + i\bar{\nu}X_A)$, we see that X_A can be neglected where ever it appears. The solution of Eq. (26) can be easily stated for the case of constant $\bar{\nu}$ and neglected gravity. It is a linear combination of two complex exponentials with exponents

$$1/2\ \left[-\hat{H} + i\bar{\nu} \pm \sqrt{4M - \bar{\nu}^2 + \hat{H}^2 + 2i\bar{\nu}(2T - \hat{H})}\right] p$$

TABLE II

$$H_A = 1 + k_2^{-2} J_{HA} + J_{NA} + k_2^{-2} \left[J_{NA} J_{HA} - J_{SA} J_{MA} + \nu^2 (J_{TA} J_{XFA} - J_{FA} J_{XTA}) \right]$$

$$X_A = k_1^{-2} J_{XTA} + \frac{k_1^{-2}}{k_2^{-2}} J_{FA} - k_1^{-2} \left[J_{SA} J_{TA} + J_{XFA} J_{MA} - J_{FA} J_{HA} - J_{XTA} J_{NA} \right]$$

$$\hat{H} = J_L - J_D + k_2^{-2}(J_H - J_{MA}) - \nu \bar{\nu} J_{FA} + k_2^{-2} \left[J_{NA} J_H + J_L J_{HA} - J_{SA} J_M - J_S J_{MA} + \nu^2 (J_{XF} J_{TA} - J_{FA} J_{XT} - J_F J_{XTA} + J_T J_{XFA}) \right]$$
$$- J_{NA}(J_D - J_g) + \nu^2 (D - J_g) k_2^{-2} (J_{TA} J_{XFA} - J_{FA} J_{XTA})$$

$$\hat{X} = 1 + k_1^{-2}(J_{XT} - J_{TA}) + \frac{k_1^{-2}}{k_2^{-2}} J_F + J_{NA} + k_1^{-2} \left[J_{XT} J_{NA} + J_{FA} J_H + J_F J_{HA} + J_L J_{XTA} - J_{XF} J_{MA} - J_{TA} J_S - J_M J_{XFA} - J_T J_{SA} \right]$$
$$- \frac{k_1^{-2}}{k_2^{-2}} J_{FA}(J_D - J_g) + \nu (D - J_g) \left[J_{FA} + k_2^{-2} (J_{FA} J_{HA} - J_{TA} J_{SA}) \right]$$

$$\hat{M} = k_2^{-2} \left[J_M + \nu^2 k_1^2 J_F + J_M J_S - J_L J_H + \nu^2 (J_F J_{XT} - J_T J_{XF}) + \nu^2 (D - J_g)(J_F J_{XTA} - J_T J_{XFA}) \right] + J_L (J_D - J_g)$$

$$\hat{T} = J_L - k_1^{-2} J_T + k_1^{-2} \left[J_F J_H - J_{XF} J_M - J_S J_T + J_L J_{XT} - \frac{k_1^{-2}}{k_2^{-2}} J_F J_A + (D - J_g)(J_F J_{HA} - J_T J_{SA}) \right]$$

$$\hat{G} = (1 + k_2^{-2} J_{HA} - i\bar{\nu} k_1^{-2} J_{XTA}) \gamma' - \left[(J_D - k_2^{-2} J_H - J_g) + i\bar{\nu}(1 + k_1^{-2} J_{XT}) \right] \gamma$$

$$E = \left[\frac{k_1^{-2}}{k_2^{-2}} J_{XFA} + k_1^{-2} (J_{XFA} J_{HA} - J_{XTA} J_{SA}) \right] \mu' + \left[\frac{k_1^{-2}}{k_2^{-2}} J_{XF} - J_{SA} + k_1^{-2} (J_{XF} J_{HA} - J_{SA} J_{XT}) + \right.$$
$$\left. i\nu \left[J_{XFA} + k_1^{-2} (J_{XT} J_{XFA} - J_{XF} J_{XTA}) \right] \right] \mu \; .$$

If we neglect the small Magnus term in $\hat{H}$, we see that the only effect of the generalized force system is the addition of $-k_2^{-2} J_{MA}$ to the real part of the coefficient of λ'. Since, in spark range work, $k_2^{-2} J_H$ is determined from this coefficient, we see that $k_2^{-2} (J_H - J_{MA})$ is the quantity actually measured. J_H appears by itself only in G which gives rise to a very small component of the yaw of repose, and for this reason no inconsistency has been noticed in spark range firings.

Two important cases where the above assumptions do not apply are those of the airship and of the torpedo. In both of these cases $\frac{\rho d^3}{m}$ is of order unity.* If the effects of drag and gravity are neglected and spin is taken to be zero, $J_N = J_L$ and Eqs. (25) reduces to

$$H_A \lambda'' + \hat{H} \lambda' - M \lambda = 0 \tag{27}$$

where $H_A = 1 + k_2^{-2} J_{HA} + J_{NA} + k_2^{-2} \left[J_{NA} J_{HA} - J_{SA} J_{MA}\right]$

$$\hat{H} = J_N + k_2^{-2} \left[J_H - J_{MA} + J_{NA} \quad J_H + J_N J_{HA} - J_{SA} J_M - J_S J_{MA}\right]$$

$$\hat{M} = M = k_2^{-2} \left[J_M(1 + J_S) - J_N J_H\right] .$$

Since J_H and J_N are positive and J_S is usually negative and less than one, ** we see that M can be negative even when K_M is positive. For the case when H_A and H are positive we see that a statically unstable configuration can be dynamically stable without spin. An explicit example of this for the airship is given on pages 110 - 112 of Ref. 11.

THE SWERVING MOTION

By the swerving motion of a missile we will mean the displacement of the center of mass due to the action of the aerodynamic force normal to the missile's axis. If we denote this non-dimensional complex

---

* Actually $\frac{m_f}{m}$, where m_f is mass of displaced fluid, is of order unity. $\frac{\rho d^3}{m}$ is approximately $\frac{1}{v}$ where v is the volume in cubic calibers(usually $3 < v < 16$)

** These statements may be verified by considering the slender body values of these coefficients given on page 26.

displacement by S

$$S = \frac{d}{m} \int_0^t \int_0^t (F_2 + iF_3)\, dt\, dt$$

$$= \int_0^p \int_0^p \Big[(-J_N + i\nu J_F)\lambda + (\nu J_{XF} + iJ_S)\mu + (-J_{NA} + i\nu J_{FA})\lambda' \qquad (28)$$

$$+ (\nu J_{XFA} + iJ_{SA})\mu' \Big]\, dp dp$$

Since the right side of Eq. (18) are essentially J terms and small, a good approximation for μ is $- i\lambda'$

$$\therefore S = \int_0^p \int_0^p \Big[(-J_N + i\nu J_F)\lambda + \Big[(J_S - J_{NA}) + i\nu(-J_{XF} + J_{FA}) \Big] \lambda' \qquad (29)$$

$$+ (J_{SA} - i\nu J_{XFA})\lambda'' \Big] \; dp\, dp$$

An examination of Eq. (26) shows that the λ'' is approximately λ multiplied by a J term and λ' is at most λ multiplied by a $J^{1/2}$ term. As yet no good measurement has been made of the coefficient of λ' and hence any determination of the coefficient of λ'' is very unlikely. If the coefficient of λ' ever is measured, it is important to note that it is $(J_S - J_{NA}) + i\nu\,(-J_{XF} + J_{FA})$ and not $J_S - i\nu J_{XF}$ as would be expected under the unmodified theory.

CENTER OF MASS TRANSFORMATIONS

An important aspect of the definitions of the aerodynamic force and moment is the dependence of the coefficients on the location of the center of mass. Although the aerodynamic force itself is independent of the location of the center of mass, the yaw λ which appears in its definition is defined in terms of the velocity of the center of mass. The dependence of the moment coefficients is more complex since they relate to the aerodynamic moment about the center of mass and are associated with λ.

We will first consider the effect on the unmodified force system of moving the center of mass a distance of q calibers along the missile axis. (Positive q will denote a movement toward the nose of the missile). All quantities relating to the missile with the new center of mass will be marked by an asterisk.

Now if corresponding points of two models of the same configuration possess the same motion, the total aerodynamic force on each model is the same and the total aerodynamic moment when computed about corresponding geometric points will be the same [2, 12, 13]. For this motion the velocity of the new c.m. with respect to the old is

$$\begin{bmatrix} u_1^* \\ u_2^* \\ u_3^* \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \times \begin{bmatrix} qd \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 + \omega_3 \; qd \\ u_3 - \omega_2 \; qd \end{bmatrix} \tag{30}$$

or $u_1^* = u_1$, (31)

$$\lambda^* = \lambda - iq\mu. \tag{32}$$

Since the angular velocity vector and the total aerodynamic force are independent of the location of the c.m.,

$$\nu^* = \nu \tag{33}$$

$$\mu^* = \mu \tag{34}$$

$$F_1^* = F_1 \tag{35}$$

$$F_2^* + iF_3^* = F_2 + iF_3 \tag{36}$$

For the moment we have

$$\begin{pmatrix} M_1^* \\ M_2^* \\ M_3^* \end{pmatrix} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} - \begin{bmatrix} qd \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 + F_3 \; qd \\ M_3 - F_2 \; qd \end{bmatrix} \tag{37}$$

or $M_1^* = M_1$ (38)

$$M_2^* + iM_3^* = M_2 + iM_3 - i(F_2 + iF_3)(qd) \tag{39}$$

Inserting the force system definitions for the unmodified system in Eqs. (35 - 39) and making use of Eqs. (31 - 34),

$$K_{DA}* = K_{DA} \qquad K_A* = K_A \tag{40}$$

$$K_N* = K_N \tag{41}$$

$$K_F* = K_F \tag{42}$$

$$K_S* = K_S - qK_N \tag{43}$$

$$K_{XF}* = K_{XF} - qK_F \tag{44}$$

$$K_M* = K_M - qK_N \tag{45}$$

$$K_T* = K_T - qK_F \tag{46}$$

$$K_H* = K_H - q(K_S + K_M) + q^2K_N \tag{47}$$

$$K_{XT}* = K_{XT} - q(K_{XF} + K_T) + q^2K_F \tag{48}$$

If we differentiate Eqs. (32) and (34), we can obtain the following transformation relations for our new variables.

$$\left[\lambda' + i\frac{\Omega_1 d}{u_1}\lambda\right]^* = \left[\lambda' + i\frac{\Omega_1 d}{u_1}\lambda\right] - iq\left[\mu' + i\frac{\Omega_1 d}{u_1}\mu\right] \tag{49}$$

$$\left[\mu' + i\frac{\Omega_1 d}{u_1}\mu\right]^* = \mu' + i\frac{\Omega_1 d}{u_1}\mu \tag{50}$$

Since Eqs. (49) and (50) are of the same form as Eqs. (32) and (34), we see that the transformation relations for the eight new ballistic coefficients should be formally the same as Eqs. (41 - 48).

$$K_{NA}* = K_{NA} \quad (51)$$

$$K_{FA}* = K_{FA} \quad (52)$$

$$K_{SA}* = K_{SA} - q\ K_{NA} \quad (53)$$

$$K_{XFA}* = K_{XFA} - q\ K_{FA} \quad (54)$$

$$K_{MA}* = K_{MA} - q\ K_{NA} \quad (55)$$

$$K_{TA}* = K_{TA} - q\ K_{FA} \quad (56)$$

$$K_{HA}* = K_{HA} - q\ (K_{SA} + K_{MA}) + q^2\ K_{NA} \quad (57)$$

$$K_{XTA}* = K_{XTA} - q\ (K_{XFA} + K_{TA}) + q^2\ K_{FA} \quad (58)$$

If the cross angular acceleration terms were neglected ($K_{SA} \equiv K_{XFA} \equiv K_{HA} \equiv K_{XTA} \equiv 0$), then the cross linear acceleration terms would have to be identically zero in order to preserve the consistency of the center of mass transformations, i.e. Eqs. (57 - 58). Thus we see the necessity for these terms involving $\mu' + i \frac{\lambda_1 d}{u_1} \mu$.

Finally it should be recalled that the only spark range technique for obtaining K_S, which has been used, is based on the measurement of values of "K_H" for two c.m. positions from the yawing motion and the use of Eq. (47). Since we now see that what is actually obtained from the yawing motions is $K_H - K_{MA}$, we add Eqs. (47) and (55) in order to find how this quantity varies with c.m. location:

$$\left[K_H - K_{MA}\right]^* = \left[K_H - K_{MA}\right] - q\left[(K_S - K_{NA}) + K_M\right] + q^2\ K_N \quad (59)$$

Hence that which is actually calculated by this spark range technique is $K_S - K_{NA}$ and not K_S.

CORRESPONDENCE WITH AERODYNAMIC COEFFICIENTS

In this section we will define aerodynamic coefficients in two coordinate systems, the standard missile-fixed and the fixed-plane, and derive the relations between them and those of our generalized ballistic coefficients.* The missile-fixed coordinates [14,17] will have force components (X, Y, Z) and moment components (L, M, N). In the fixed-plane coordinates we will designate the force components as $(X, \tilde{Y}, \tilde{Z})$ and the moment components as $(\tilde{L}, \tilde{M}, \tilde{N})$. The axial components for both systems are the same and are defined as

*It should be emphasized that a simple linear expansion of force and moment in the fixed-plane coordinates is only possible for a symmetric missile. This can be seen from an examination of Eqs. (A8-9) of the appendix.

$$X = (1/2)\rho V^2 S C_X \doteq -(\tfrac{1}{2})\rho V^2 S C_D \qquad (60)$$

$$L = (1/2)\rho V^2 S \ell (\tfrac{p\ell}{2V}) C_{\ell_p} \qquad (61)$$

where V is the total velocity

S is a reference area

ℓ is a reference length

p is axial component of angular velocity ($\therefore p = \omega_1$)

If we limit ourselves to small yaw where $V \doteq u_1$, we see from comparing Eqs. (60 - 61) with Eqs. (1) and (3) that

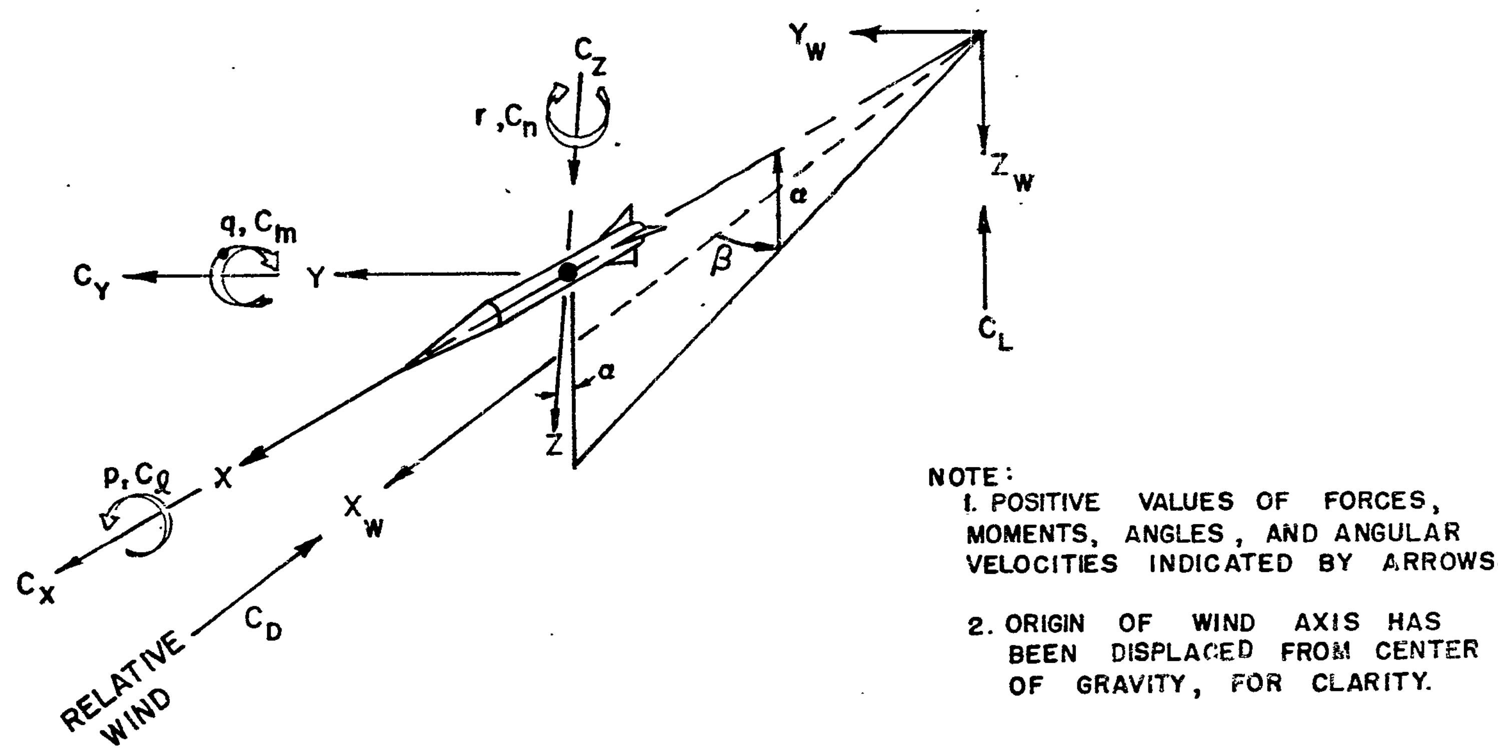
$$K_{DA} = -1/2 \frac{S}{d^2} C_X \doteq 1/2 \frac{S}{d^2} C_D \qquad (62)$$

$$K_A = -1/4 \frac{S\ell^2}{d^4} C_{\ell_p} \qquad (63)$$

For convenience in comparing the aerodynamic coefficients with the ballistic, we will expand the transverse force and moment in complex form. Since it can be shown that for a missile possessing trigonal or greater rotational symmetry, pairs of aerodynamic coefficients are related, we will explicitly indicate such equivalent sets in this expansion by the symbols $(C_Y, \pm C_Z)$ and $(C_m, \pm C_n)$ which stand for either one of the quantities inside. For example, since $C_{Y_\beta} = C_{Z_\alpha}$ and $C_{Y_{p\alpha}} = -C_{Z_{p\beta}}$, the symbols $(C_{Y_\beta}, C_{Z_\alpha})$ and $(C_{Y_{p\alpha}}, -C_{Z_{p\beta}})$ will be used to stand for either one of the two equivalent quantities in the parentheses. With this in mind the transverse force and moment* acting on a symmetric missile may be written as:

$$\begin{aligned} Y + iZ = (\tfrac{1}{2})\rho V^2 S \Bigg\{ & \left[(C_{Y_\beta}, C_{Z_\alpha}) - i(\tfrac{p\ell}{2V})(C_{Y_{p\alpha}}, -C_{Z_{p\beta}})\right](\beta + i\alpha) \\ & + \left[(\tfrac{p\ell}{2V})(C_{Y_{pq}}, C_{Z_{p_r}}) - i(C_{Y_r}, -C_{Z_q})\right]\frac{(q + ir)\ell}{2V} \\ & + \left[(C_{Y_{\dot\beta}}, C_{Z_{\dot\alpha}}) - i(\tfrac{p\ell}{2V})(C_{Y_{p\dot\alpha}}, -C_{Z_{p\dot\beta}})\right]\frac{(\dot\beta + i\dot\alpha)\ell}{2V} \\ & + \left[(\tfrac{p\ell}{2V})(C_{Y_{p\dot q}}, C_{Z_{p\dot r}}) - i(C_{Y_{\dot r}}, -C_{Z_{\dot q}})\right]\frac{(\dot q + i\dot r)\ell^2}{4V^2}\Bigg\} \end{aligned} \qquad (64)$$

* In Figure 1, which is based on Figure 2 of Reference 17, the various forces, moments, angles, and angular velocities are shown.



C_Z
r, C_n
Y_W
Z_W
C_L
q, C_m
C_Y
Y
α
β
α
Z
X
X_W
p, C_ℓ
C_X
C_D
RELATIVE WIND
NOTE:
1. POSITIVE VALUES OF FORCES, MOMENTS, ANGLES, AND ANGULAR VELOCITIES INDICATED BY ARROWS.
2. ORIGIN OF WIND AXIS HAS BEEN DISPLACED FROM CENTER OF GRAVITY, FOR CLARITY.

Fig. 1

$$M + iN = \left(\frac{1}{2}\right) \rho V^2 S\ell \left\{ \left[\left(\frac{p\ell}{2V}\right) (C_{m_{p\beta}}, C_{n_{p\alpha}}) - i(C_{m_\alpha}, - C_{n_\beta}) \right] (\beta + i\alpha) \right.$$

$$+ \left[(C_{m_q}, C_{n_r}) - i\left(\frac{p\ell}{2V}\right)(C_{m_{pr}}, - C_{n_{pq}}) \right] \frac{(q + ir)\ell}{2V}$$

$$+ \left[\left(\frac{p\ell}{2V}\right)(C_{m_{p\dot\beta}}, C_{n_{p\dot\alpha}}) - i(C_{m_{\dot\alpha}}, - C_{n_{\dot\beta}}) \right] \frac{(\dot\beta + i\dot\alpha)\ell}{2V} \qquad (65)$$

$$\left. + \left[(C_{m_{\dot q}}, C_{n_{\dot r}}) - i\left(\frac{p\ell}{2V}\right)(C_{m_{p\dot r}}, - C_{n_{p\dot q}}) \right] \frac{(\dot q + i\dot r)\ell^2}{4V^2} \right\}$$

where α, β are angles of attack and sideslip in missile-fixed coordinates
q, r are transverse components of angular velocity in the missile-fixed coordinates.

In the fixed-plane coordinates the expansion of $\tilde{Y} + i\tilde{Z}$ and $\tilde{M} + i\tilde{N}$ would be exactly that of Eqs. (64 - 65) with the tilde superscripts inserted. If we set $\Omega_1 = \omega_1$ in Eqs. (15 - 16) and compare with Eqs. (64 - 65) and neglect the small $\frac{u_1'}{u_1}$, we obtain the following relations for the missile fixed aerodynamic coefficients.

$$(\tfrac{1}{2})S\, C_{Y_\beta} = (\tfrac{1}{2})S\, C_{Z_\alpha} = (-K_N - \nu^2 K_{FA})d^2$$

$$(\tfrac{\ell}{4})S\, C_{Y_r} = -(\tfrac{\ell}{4})S\, C_{Z_q} = (-K_S - \nu^2 K_{XFA})d^3$$

$$(\tfrac{\ell}{4})S\, C_{Y_{\dot\beta}} = (\tfrac{\ell}{4})S\, C_{Z_{\dot\alpha}} = -K_{NA}\, d^3$$

$$(\tfrac{\ell^2}{8})S\, C_{Y_{\dot r}} = -(\tfrac{\ell^2}{8})S\, C_{Z_{\dot q}} = -K_{SA}\, d^4$$

$$(\tfrac{\ell}{4})\, S\, C_{Y_{p\alpha}} = -(\tfrac{\ell}{4})S\, C_{Z_{p\beta}} = (-K_F + K_{NA})d^3$$

$$(\tfrac{\ell^2}{8})S\, C_{Y_{pq}} = (\tfrac{\ell^2}{8})S\, C_{Z_{pr}} = (K_{XF} - K_{SA})d^4$$

$$(\tfrac{\ell^2}{8})S\, C_{Y_{p\dot\alpha}} = -(\tfrac{\ell^2}{8})S\, C_{Z_{p\dot\beta}} = -K_{FA}\, d^4$$

$$(\tfrac{\ell^3}{16})\, S\, C_{Y_{p\dot q}} = (\tfrac{\ell^3}{16})S\, C_{Z_{p\dot r}} = K_{XFA}\, d^5 \qquad (66)$$

$$(\tfrac{\ell}{2})S\, C_{m_\alpha} = -(\tfrac{\ell}{2})S\, C_{n_\beta} = (K_M + \nu^2 K_{TA})d^3$$

$$(\tfrac{\ell^2}{4})S\, C_{m_q} = (\tfrac{\ell^2}{4})S\, C_{n_r} = (-K_H - \nu^2 K_{XTA})d^4$$

$$(\tfrac{\ell^2}{4})S\, C_{m_{\dot\alpha}} = -(\tfrac{\ell^2}{4})S\, C_{n_{\dot\beta}} = K_{MA}\, d^4$$

$$(\tfrac{\ell^3}{8})S\, C_{m_{\dot q}} = (\tfrac{\ell^3}{8})S\, C_{n_{\dot r}} = -K_{HA}\, d^5$$

$$(\tfrac{\ell^2}{4})S\, C_{m_{p\beta}} = (\tfrac{\ell^2}{4})S\, C_{n_{p\alpha}} = (-K_T + K_{MA})d^4$$

$$(\tfrac{\ell^3}{8})S\, C_{m_{pr}} = -(\tfrac{\ell^3}{8})S\, C_{n_{pq}} = (-K_{XT} + K_{HA})d^5$$

$$(\tfrac{\ell^3}{8})S\, C_{m_{p\dot\beta}} = (\tfrac{\ell^3}{8})S\, C_{n_{p\dot\alpha}} = -K_{TA}\, d^5$$

$$(\tfrac{\ell^4}{16})S\, C_{m_{p\dot r}} = -(\tfrac{\ell^4}{16})S\, C_{n_{p\dot q}} = -K_{XTA}\, d^6$$

In order to connect our ballistic coefficients to the aerodynamic coefficients in the fixed plane coordinates we need only equate Ω_1 to zero in Eqs. (15 - 16), insert the tilde superscript in Eqs. (64 - 65) and compare. Doing this and solving for the ballistic coefficients,*

$$
\begin{aligned}
K_N &= -\left(\tfrac{1}{2}\right)\left(\tfrac{S}{d^2}\right)C_{\tilde{Y}_\beta} = -\left(\tfrac{1}{2}\right)\left(\tfrac{S}{d^2}\right)C_{\tilde{Z}_\alpha} & K_M &= \left(\tfrac{1}{2}\right)\left(\tfrac{S\ell}{d^3}\right)C_{\tilde{m}_\alpha} = -\left(\tfrac{1}{2}\right)\left(\tfrac{S\ell}{d^3}\right)C_{\tilde{n}_\beta} \\
K_S &= -\left(\tfrac{1}{4}\right)\left(\tfrac{S\ell}{d^3}\right)C_{\tilde{Y}_r} = \left(\tfrac{1}{4}\right)\left(\tfrac{S\ell}{d^3}\right)C_{\tilde{Z}_q} & K_H &= -\left(\tfrac{1}{4}\right)\left(\tfrac{S\ell^2}{d^4}\right)C_{\tilde{m}_q} = -\left(\tfrac{1}{4}\right)\left(\tfrac{S\ell^2}{d^4}\right)C_{\tilde{n}_r} \\
K_{NA} &= -\left(\tfrac{1}{4}\right)\left(\tfrac{S\ell}{d^3}\right)C_{\tilde{Y}_{\dot\beta}} = -\left(\tfrac{1}{4}\right)\left(\tfrac{S\ell}{d^3}\right)C_{\tilde{Z}_{\dot\alpha}} & K_{MA} &= \left(\tfrac{1}{4}\right)\left(\tfrac{S\ell^2}{d^4}\right)C_{\tilde{m}_{\dot\alpha}} = -\left(\tfrac{1}{4}\right)\left(\tfrac{S\ell^2}{d^4}\right)C_{\tilde{n}_{\dot\beta}} \\
K_{SA} &= -\left(\tfrac{1}{8}\right)\left(\tfrac{S\ell^2}{d^4}\right)C_{\tilde{Y}_{\dot r}} = \left(\tfrac{1}{8}\right)\left(\tfrac{S\ell^2}{d^4}\right)C_{\tilde{Z}_{\dot q}} & K_{HA} &= -\left(\tfrac{1}{8}\right)\left(\tfrac{S\ell^3}{d^5}\right)C_{\tilde{m}_{\dot q}} = -\left(\tfrac{1}{8}\right)\left(\tfrac{S\ell^3}{d^5}\right)C_{\tilde{m}_{\dot r}} \\
K_F &= -\left(\tfrac{1}{4}\right)\left(\tfrac{S\ell}{d^3}\right)C_{\tilde{Y}_{p\alpha}} = \left(\tfrac{1}{4}\right)\left(\tfrac{S\ell}{d^3}\right)C_{\tilde{Z}_{p\beta}} & K_T &= -\left(\tfrac{1}{4}\right)\left(\tfrac{S\ell^2}{d^4}\right)C_{\tilde{m}_{p\beta}} = -\left(\tfrac{1}{4}\right)\left(\tfrac{S\ell^2}{d^4}\right)C_{\tilde{n}_{p\alpha}} \qquad (67) \\
K_{XF} &= \left(\tfrac{1}{8}\right)\left(\tfrac{S\ell^2}{d^4}\right)C_{\tilde{Y}_{pq}} = \left(\tfrac{1}{8}\right)\left(\tfrac{S\ell^2}{d^4}\right)C_{\tilde{Z}_{pr}} & K_{XT} &= -\left(\tfrac{1}{8}\right)\left(\tfrac{S\ell^3}{d^5}\right)C_{\tilde{m}_{pr}} = \left(\tfrac{1}{8}\right)\left(\tfrac{S\ell^3}{d^5}\right)C_{\tilde{n}_{pq}} \\
K_{FA} &= -\left(\tfrac{1}{8}\right)\left(\tfrac{S\ell^2}{d^4}\right)C_{\tilde{Y}_{\dot{p\alpha}}} = \left(\tfrac{1}{8}\right)\left(\tfrac{S\ell^2}{d^4}\right)C_{\tilde{Z}_{\dot{p\beta}}} & K_{TA} &= -\left(\tfrac{1}{8}\right)\left(\tfrac{S\ell^3}{d^5}\right)C_{\tilde{m}_{\dot{p\beta}}} = -\left(\tfrac{1}{8}\right)\left(\tfrac{S\ell^3}{d^5}\right)C_{\tilde{n}_{\dot{p\alpha}}} \\
K_{XFA} &= \left(\tfrac{1}{16}\right)\left(\tfrac{S\ell^3}{d^5}\right)C_{\tilde{Y}_{\dot{pq}}} = \left(\tfrac{1}{16}\right)\left(\tfrac{S\ell^3}{d^5}\right)C_{\tilde{Z}_{\dot{pr}}} & K_{XTA} &= -\left(\tfrac{1}{16}\right)\left(\tfrac{S\ell^4}{d^6}\right)C_{\tilde{m}_{\dot{pr}}} = \left(\tfrac{1}{16}\right)\left(\tfrac{S\ell^4}{d^6}\right)C_{\tilde{n}_{\dot{pq}}}
\end{aligned}
$$

Finally we combine Eqs. (66) and (67) to obtain the relations between the two different sets of aerodynamic coefficients.**

* The suggested aeroballistic C_N's of Ref. 15 and 16 are the same as $C_{\tilde{Z}}$'s while the aeroballistic C_M's are the same as $C_{\tilde{m}}$'s.

** It is interesting to note that Ref. 15 makes the implicit assumption that coefficients in missile-fixed and fixed-plane coordinates are equal. Since the aeroballistic C_N's and C_m's of that report are actually in the fixed-plane coordinates, its results are unaffected.

$$
\begin{aligned}
C_{Y_\beta} &= C_{\tilde{Y}_\beta} + \left(\frac{p\ell}{2V}\right)^2 C_{\tilde{Y}_{p\alpha}} & C_{m_\alpha} &= C_{\tilde{m}_\alpha} - \left(\frac{p\ell}{2V}\right)^2 C_{\tilde{m}_{p\beta}} \\
C_{Y_r} &= C_{\tilde{Y}_r} - \left(\frac{p\ell}{2V}\right)^2 C_{\tilde{Y}_{pq}} & C_{m_q} &= C_{\tilde{m}_q} + \left(\frac{p\ell}{2V}\right)^2 C_{\tilde{m}_{pr}} \\
C_{Y_{\dot\beta}} &= C_{\tilde{Y}_{\dot\beta}} & C_{m_{\dot\alpha}} &= C_{\tilde{m}_{\dot\alpha}} \\
C_{Y_{\dot r}} &= C_{\tilde{Y}_{\dot r}} & C_{m_{\dot q}} &= C_{\tilde{m}_{\dot q}} \\
C_{Y_{p\alpha}} &= C_{\tilde{Y}_{p\alpha}} - C_{\tilde{Y}_{\dot\beta}} & C_{m_{p\beta}} &= C_{\tilde{m}_{p\beta}} + C_{\tilde{m}_{\dot\alpha}} \qquad (68) \\
C_{Y_{pq}} &= C_{\tilde{Y}_{pq}} + C_{\tilde{Y}_{\dot r}} & C_{m_{pr}} &= C_{\tilde{m}_{pr}} - C_{\tilde{m}_{\dot q}} \\
C_{Y_{p\dot\alpha}} &= C_{\tilde{Y}_{p\dot\alpha}} & C_{m_{p\dot\beta}} &= C_{\tilde{m}_{p\dot\beta}} \\
C_{Y_{p\dot q}} &= C_{\tilde{Y}_{p\dot q}} & C_{m_{p\dot r}} &= C_{\tilde{m}_{p\dot r}}
\end{aligned}
$$

A similar set of relations apply to the Z and N components. The only difference is that those equations with pairs of coefficients on the right side would have the sign between changed.

At this point we should indicate a third possible choice of coordinate system for aerodynamic coefficients. This would be a missile-fixed coordinate system where fixed-plane derivatives are used in the expansion. The definitions for these coordinates could be obtained if $(\beta + i\alpha)$ were replaced by $\dot\beta + i\dot\alpha + ip(\beta + i\alpha)$ in Eqs. (64-65) and $\dot q + i\dot r$ were replaced by $\dot q + i\dot r + ip(q + ir)$ in the same two equations. It can easily be shown that the resulting coefficients are equal to the fixed-plane coefficients* and hence, have a one-to-one correspondence with our generalized ballistic K_i's.

SLENDER BODY VALUES

In Ref. 8 values of all non-Magnus coefficients of slender bodies with arbitrary cross-section are given and in Ref. 9 the results for bodies of revolution are derived by a simple technique. Since one of the valuable features of our generalized ballistic system is its one-to-one correspondence with the force system which arises from theoretical flow calculations, we state the results of Ref. 9 in terms of our generalized ballistic coefficients

*This is only true, however, for the small angles where the non-rolling coordinates are the same as the fixed-plane coordinate. The above definitions actually correspond to non-rolling derivatives and are exactly equivalent to our selection for new ballistic variables.

$$K_N = s_b \qquad K_M = -s_b \hat{x} + v$$

$$K_S = -s_b \hat{x} \qquad K_H = s_b \hat{x}^2 - v(\hat{x} - \hat{x}_c)$$

$$K_{NA} = v \qquad K_{MA} = -v(\hat{x} - \hat{x}_c) \tag{69}$$

$$K_{SA} = -v(\hat{x} - \hat{x}_c) \qquad K_{HA} = v\bar{k}^2 = v(\hat{x}^2 - 2\hat{x}_c\hat{x} + \bar{k}_b^2)$$

where

s_b is area of base in calibers squared

$\hat{x}$ is distance from base to c.m. in calibers

v is volume in calibers cubed

$\hat{x}_c$ is distance from base to centroid in calibers

$\bar{k}$ is transverse radius of gyration of a homogeneous model about the c.m. in calibers

$\bar{k}_b$ is transverse radius of gyration of a homogeneous model about the base in calibers

By use of Eqs. (69) we can estimate all of the coefficients which appear in the equation of yawing motion for an airship (Eq. 27).

SUMMARY

1. The Ballistic Theory of Motion has been extended to include all linear terms arising from the linear and angular acceleration of the missile.

2. The Aerodynamic Theory of Motion has been extended and the original inconsistencies have been corrected.

3. A unique correspondence between ballistic and aerodynamic coefficients has been made and the differences between force expansions for symmetric missiles in missile-fixed and fixed-plane coordinates have been obtained.

4. Important results of this analysis are a better interpretation of free flight measurements and an opportunity to compare theoretical computations of coefficients with free flight values.

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APPENDIX A

Expansion of Transverse Force and Moment For Missile With Only Mirror Symmetry

The usual linear force assumption for a missile with no specified symmetry is the assumption that in missile-fixed coordinates the aerodynamic force and moment are linear functions of

$$\frac{u_2}{u_1}, \frac{u_3}{u_1}, \frac{\omega_2 d}{u_1}, \frac{\omega_3 d}{u_1} .$$

These components are then replaced by combinations of λ, $\bar{\lambda}$, μ and $\bar{\mu}$, and the assumption of trigonal or greater rotational symmetry requires that the coefficients of $\bar{\lambda}$ and $\bar{\mu}$ vanish. This plus mirror symmetry provides us with the simple definitions given by Eqs. (1 - 4). (See Refs. 1, 2, 4.)

In this appendix we will obtain the vector expansion of the generalized transverse force and moment for a missile possessing only mirror symmetry. This means that we will have terms in $\bar{\lambda}$, $\bar{\mu}$, $\lambda' - i\frac{\Omega_1 d}{u_1}\bar{\lambda}$ and $\bar{\mu}' - i\frac{\Omega_1 d}{u_1}\bar{\mu}$.

This necessity for stating our expansion with respect to the missile-fixed coordinates, however, does not conflict with our preference for taking derivatives in the non-rolling system. We will consider in detail the yaw term in the expansion of the transverse force and apply our results to the general expansion of both force and moment.*

The contribution of the yaw to the dimensionless transverse force can be written in the following form in missile-fixed coordinates:

$$f_{2m} + if_{3m} = b_0 + b_1\lambda_m + b_2\bar{\lambda}_m \qquad (A1)$$

where b_0, b_1, b_2 are complex functions of Mach number, axial spin, missile shape and fluid properties.

In order to obtain the dependence of these coefficients on spin, we consider the implications of mirror symmetry. If we locate the 1-2 plane in the plane of mirror symmetry, then a missile possessing mirror symmetry will be invariant under a reversal of the 3 axis. Hence the functional dependence of the force and moment on the dynamic variables measured in the new coordinate system will be the same. Therefore $f_2^* + if_3^*$ are measured in the transformed coordinate system

$$f_{2m}^* + if_{3m}^* = b_0(\nu^*) + b_1(\nu^*)\lambda_m^* + b_2(\nu^*)\bar{\lambda}_m^* \qquad (A2)$$

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* The reasoning for the non-generalized force system is given in detail in Ref. 4.

But we know that

$$f_{2m}^* + if_{3m}^* = f_{2m} - if_{3m} = \overline{f_{2m} + if_{3m}} \tag{A3}$$

$$\lambda_m^* = \overline{\lambda}_m$$

$$\nu^* = -\nu$$

$$\therefore \quad b_0(-\nu) + b_1(-\nu)\,\overline{\lambda}_m + b_2(-\nu)\lambda_m = \overline{b_0(\nu) + b_1(\nu)\lambda_m + b_2(\nu)\overline{\lambda}_m} \tag{A4}$$

or

$$\overline{b}_0(\nu) = b_0(-\nu)$$

$$\overline{b}_1(\nu) = b_1(-\nu)$$

$$\overline{b}_2(\nu) = b_2(-\nu)$$

Thus we see that the real parts of b_0, b_1, and b_2 are even functions of ν and the imaginary parts are odd functions of ν. With this in mind we can make the following definition:

$$f_{2m} + if_{3m} = (-K_{N_0} + i\nu K_{F_0}) + (-K_N + i\nu K_F)\lambda_m + (-\hat{K}_N + i\nu\hat{K}_F)\overline{\lambda}_m \tag{A5}$$

where all the K_i's are even functions of ν.

We now obtain the vector form of Eq. (A5). If θ is the angle between the 2-axis in the missile-fixed system and an arbitrary 2-axis,*

$$f_2 + if_3 = (f_{2m} + if_{3m})\, e^{i\theta} \tag{A6}$$

$$\lambda = \lambda_m e^{i\theta}$$

$$\overline{\lambda} = \overline{\lambda}_m e^{-i\theta}$$

$$\therefore \quad f_2 + if_3 = (-K_{N_0} + i\nu K_{F_0})e^{i\theta} + (-K_N + i\nu K_F)\lambda + (-\hat{K}_N + i\nu\hat{K}_F)\overline{\lambda}e^{2i\theta} \tag{A7}$$

*Since the missile-fixed 2 axis is in the plane of mirror symmetry, θ is angle between this plane and the arbitrary 2 axis.

From this discussion we can make these observations:

(1) Since 1 and λ transform similarly under reversal of the 3 axis, the coefficient of 1, which is the constant term, and the coefficient of λ have the same mirror symmetry properties.

(2) Since a variable and its conjugate transform alike under the mirror transformation, their coefficients have the same properties.

(3) In the vector expansion of force or moment the coefficient of 1 should have an $e^{i\theta}$ factor and the coefficient of a conjugate variable should have a $e^{2i\theta}$ factor.

With these points in mind we can now write the expansion of transverse aerodynamic force and moment for a missile possessing a plane of mirror symmetry.* From an examination of Eqs. (15) and (16) it follows that:

$$\begin{aligned} F_2 + iF_3 = (\rho d^2 u_1{}^2) \Big\{ & (-K_{N_0} + i\nu K_{F_0}) e^{i\theta} + \Big[(-K_N + i\nu K_F)\lambda \\ & + (\nu K_{XF} + iK_S)\mu + (-K_{NA} + i\nu K_{FA})\left(\lambda' + i\frac{\Omega_1 d}{u_1}\lambda\right) \\ & + (\nu K_{XFA} + iK_{SA})\left(\mu' + i\frac{\Omega_1 d}{u_1}\mu\right)\Big] \\ & + \Big[(-\hat{K}_N + i\nu\hat{K}_F)\bar{\lambda} + (\nu\hat{K}_{XF} + i\hat{K}_S)\bar{\mu} \\ & + (-\hat{K}_{NA} + i\nu\hat{K}_{FA})\left(\bar{\lambda}' - i\frac{\Omega_1 d}{u_1}\bar{\lambda}\right) + (\nu\hat{K}_{XFA} + i\hat{K}_{SA})\left(\bar{\mu}' - i\frac{\Omega_1 d}{u_1}\bar{\mu}\right)\Big] e^{2i\theta}\Big\} \end{aligned} \tag{A8}$$

$$\begin{aligned} M_2 + iM_3 = \rho d^3 u_1{}^2 \Big\{ & (-\nu K_{T_0} - iK_{M_0}) e^{i\theta} + \Big[(-\nu K_T - iK_M)\lambda + (-K_H + i\nu K_{XT})\mu \\ & + (-\nu K_{TA} - iK_{MA})\left(\lambda' + i\frac{\Omega_1 d}{u_1}\lambda\right) + (-K_{HA} + i\nu K_{XTA})\left(\mu' + i\frac{\Omega_1 d}{u_1}\mu\right)\Big] \\ & + \Big[(-\nu\hat{K}_T - i\hat{K}_M)\bar{\lambda} + (-\hat{K}_H + i\nu\hat{K}_{XT})\bar{\mu} + (-\nu\hat{K}_{TA} - i\hat{K}_{MA})\left(\bar{\lambda}' - i\frac{\Omega_1 d}{u_1}\bar{\lambda}\right) \\ & + (-\hat{K}_{HA} + i\nu\hat{K}_{XTA})\left(\bar{\mu}' - i\frac{\Omega_1 d}{u_1}\bar{\mu}\right)\Big] e^{2i\theta}\Big\} \end{aligned} \tag{A9}$$

where all K_i's are even functions of ν. Finally if we want to drop the condition of mirror symmetry, we need only say that the non-Magnus coefficients K_N, K_S, etc. and the products of Magnus coefficients by ν, νK_F, νK_{XF}, etc, are arbitrary functions of spin.**

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*In Ref. 15, all coefficients of conjugates and the Magnus term K_{FO} and K_{TO} are neglected and the resulting equations of motion are solved. In the symbols of Ref. 18, $K_{NO} \equiv -K_N\lambda_\epsilon$ and $K_{MO} \equiv -K_M\lambda_\epsilon$ where λ_ϵ is the asymmetry angle.

**In Ref. 19, Hazeltine develops the dynamic theory for a missile with rotational symmetry but lacking mirror symmetry.

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